

## Determination of Resultant Forces

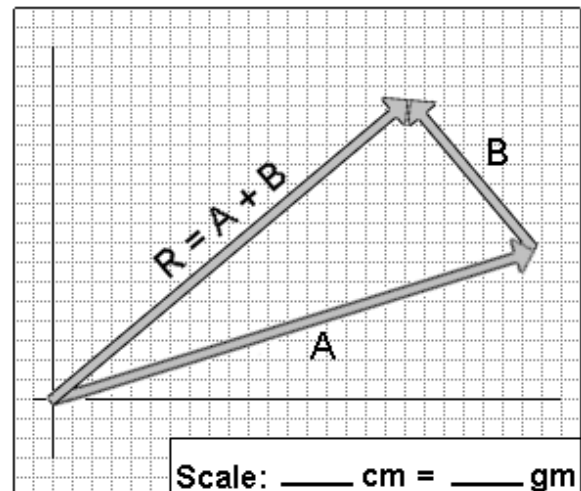
### Introduction

If several forces, each having different magnitudes and directions, act on a point object, their net effect can be represented by a single force called a **resultant** force. The magnitude and direction of the resultant force can be found using a special process known as **vector addition**. Vector addition can be done both analytically and graphically. In addition, resultant force vectors can be found experimentally.

### Discussion

In the graphical technique of vector addition, each force is represented by an arrow whose length is proportional to the vector's magnitude. Each arrow is oriented at an angle  $\theta$  that corresponds to the direction in which the force acts. In this technique, the tail of each arrow is connected to the head of the previous arrow until all forces have been drawn. A single arrow is then drawn from the tail of the first arrow (at the coordinate system origin) to the tip of the last arrow. This final arrow then represents the resultant force. Its length is proportional to the magnitude of the resultant vector and its angle  $\theta$  corresponds to the direction in which the resultant force acts.

The addition of two vectors **A** and **B** is shown in the diagram at the right. Using the "head-to-tail" method, we first draw vector **A** by placing its tail at the origin of the coordinate system. Next we draw vector **B** by placing its tail at the tip of vector **A**. The resultant vector  $\mathbf{R} = \mathbf{A} + \mathbf{B}$  is found by drawing a straight line from the tail of vector **A** to the tip of vector **B**. The magnitude of **R** is proportional to its length and the direction in which it acts is the angle it makes with the horizontal. (Note: It doesn't matter which vector is drawn first. The result will be the same.)



In the analytical process of vector addition, each vector to be added is first broken down into its x and y components. The components along each axis are then added algebraically to obtain the components of the resultant vector along each axis. This leads to the following:

$$R_x = A_x + B_x = A\cos\theta_1 + B\cos\theta_2$$

$$R_y = A_y + B_y = A\sin\theta_1 + B\sin\theta_2$$

The magnitude of the resultant vector is then found using the Pythagorean Theorem

$$|R| = \sqrt{R_x^2 + R_y^2}$$

Furthermore, the angle  $\theta$  that the resultant  $\mathbf{R}$  makes with the x axis is given by the following:

$$\theta = \arctan \frac{R_y}{R_x} = \tan^{-1} \frac{R_y}{R_x}$$

Since a nonzero resultant force accelerates an object, another force must be applied to establish equilibrium. If  $F_A$  and  $F_B$  are two known forces (represented by vectors  $\mathbf{A}$  and  $\mathbf{B}$ ) applied to an object, they will have a resultant force (represented by the vector  $\mathbf{R}$ ). A force equal in magnitude and opposite in direction to  $\mathbf{R}$  must be applied to keep the object in equilibrium. The force needed to establish equilibrium is called the **equilibrant force**.

The resultant force  $\mathbf{R}$  is found experimentally by first finding the equilibrant force  $\mathbf{E}$  needed to balance all the individual forces. The resultant force is then the force having the same magnitude as the equilibrant force  $\mathbf{E}$  but acting in the opposite direction. A device known as a **force table** is used find the resultant force experimentally.

A force table is a disc whose outer rim is graduated in degrees. Strings attached to a metal ring in the center pass over pulleys mounted at the edge of the disc. The angular positions of these pulleys may be adjusted.



## Objective

The purpose of this lab is to gain experience in working with vector quantities. The lab demonstrates the process of combining several vectors using various vector addition techniques to form a single resultant vector. Experimental, graphical, and analytical methods will be used to determine the resultant of several vector combinations.

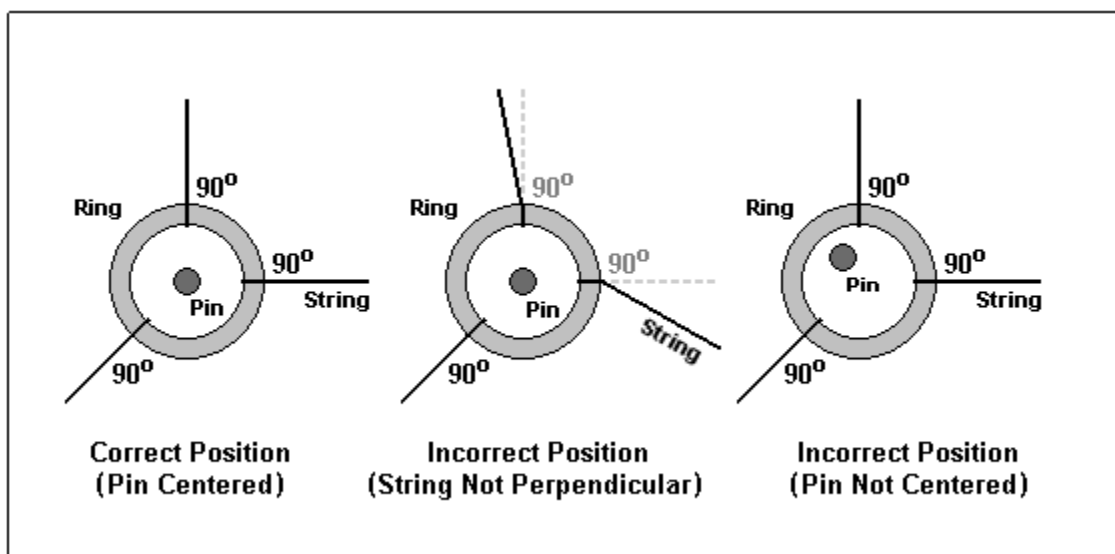
## Equipment

- force table
- weights
- protractor
- measuring scale
- calculator
- bubble level

## Procedure

A force table will be used to experimentally determine the resultant of several forces acting on a ring located at the center of the table. Strings attached to the ring pull it in different directions. The magnitude of each pull and its direction can be varied by adjusting the masses attached to the strings and the location of the pulleys on the rim of the table. **For best results, the force table should be level and the pulleys should be mounted perpendicular to the rim of the table. When in equilibrium, the ring will remain centered on the table pin.**

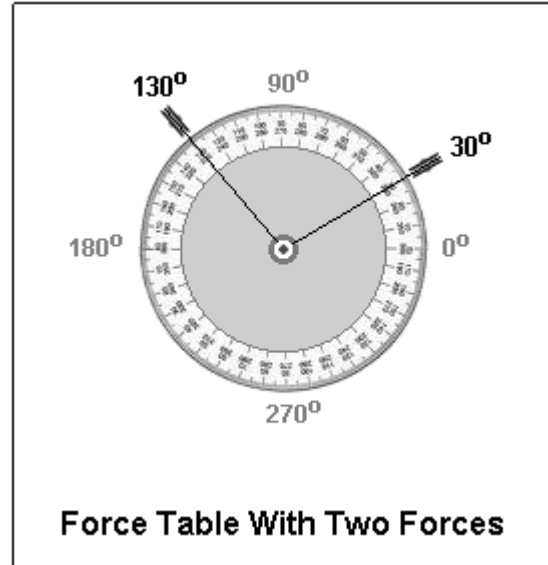
Note: The ring is in equilibrium when it is centered on the force table pin. Be sure that all the strings are positioned so that they are perpendicular to the ring as shown below.



## Experimental Determination of the Resultant of Two Forces

### Part 1: Experimental Method

1. Place a pulley at the  $0^\circ$  mark on the force table and place a total of 120 gm on the end of the string (don't forget to include the mass of the hanger!). Place a second pulley at the  $90^\circ$  mark and place a total of 80 gm on the end of that string (also including the mass of the hanger!).
2. By trial and error, determine the angle at which a balancing force must act in order to place the ring in equilibrium. Place a pulley at this angle, pass a string over the pulley, attach a hanger to end of the string, and add mass disks until the ring is in equilibrium.



3. Record the experimental angle and amount of mass (don't forget to include the hanger) needed to balance the other two forces in the data table below. (This is the equilibrant force **E** that balances the resultant force **R**). Now determine and record the resultant force **R** based on your experimental results for the equilibrant force **E**. (The resultant force will be equal in magnitude to the equilibrant, but in the opposite direction.)

| Experimental Determination |       |      |  |
|----------------------------|-------|------|--|
| Force                      | Angle | Mass |  |
| 1                          |       |      |  |
| 2                          |       |      |  |
| Equilibrant                |       |      |  |
| Resultant                  |       |      |  |

## Part 1: Graphical Method

Using your graph paper, a protractor, and a centimeter scale, draw arrows to represent the forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$  described above. Choose a scale and draw each force vector so that its length is proportional to the magnitude of the force. The direction of each arrow must be the same direction as the force it represents. Using the head-to-tail method, draw an arrow that represents the resultant of the two vectors. (Don't forget to record the scale you chose.)

From the length of the resultant vector  $\mathbf{R}$ , determine its magnitude according to your scale. Using the protractor, determine the resultant vector's direction (angle). Record your results in the table below.

| Graphical Determination |       |      |        |
|-------------------------|-------|------|--------|
| Force                   | Angle | Mass | Length |
| 1                       |       |      |        |
| 2                       |       |      |        |
| Resultant               |       |      |        |

## Part 1: Component Method

Using your calculator, determine the x and y components of  $\mathbf{F}_1$  and  $\mathbf{F}_2$ . Remember that  $F_x = F \cos \theta$  and  $F_y = F \sin \theta$ . Find the x and y components of the resultant from the sum of x and y components. Calculate the magnitude of the resultant force from the square root of  $(R_x^2 + R_y^2)$ . Calculate the direction of the resultant force from  $\theta_R = \tan^{-1}(R_y/R_x)$ . Does this result agree with the graphical method?

| Component Determination |           |           |               |               |
|-------------------------|-----------|-----------|---------------|---------------|
| Vector                  | Magnitude | Direction | X - Component | Y - Component |
| 1                       |           |           |               |               |
| 2                       |           |           |               |               |
|                         |           |           | $R_x =$       | $R_y =$       |

Resultant  $R =$    $\theta_R =$

## Part 1: Error Determination

Calculate the percent error of the the experimental value of  $F_R$  compared to the calculated value of  $F_R$  (the accepted value). Also calculate the percent error of the graphical solution of  $F_R$  compared to the calculated solution.

$$\text{Experimental Error} = \left| \frac{\text{experimental value} - \text{calculated value}}{\text{calculated value}} \right| * 100\%$$

$$\text{Graphical Error} = \left| \frac{\text{graphical value} - \text{calculated value}}{\text{calculated value}} \right| * 100\%$$

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## Parts 2, 3, and 4

Repeat the steps in Part 1 to determine the resultant of the three additional force combinations (2, 3, and 4) shown below using the (1) experimental, (2) graphical, and (3) component methods for each combination of force vectors. (Don't forget to include the percent error for each.)

| Force Combinations |        |           |           |        |           |           |
|--------------------|--------|-----------|-----------|--------|-----------|-----------|
|                    | Vector | Magnitude | Direction | Vector | Magnitude | Direction |
| 2                  | A      | 175 gm    | 60°       | B      | 130 gm    | 160°      |
| 3                  | A      | 110 gm    | 145°      | B      | 140 gm    | 230°      |
| 4                  | A      | 150 gm    | 45°       | B      | 180 gm    | 290°      |

## *Part 2: Experimental Method*

Using the force table, simulate the forces described in Part 2 and experimentally determine the magnitude and direction of their resultant. Record your results in the table below.

| Experimental Determination |       |      |  |
|----------------------------|-------|------|--|
| Force                      | Angle | Mass |  |
| 1                          |       |      |  |
| 2                          |       |      |  |
| Equilibrant                |       |      |  |
| Resultant                  |       |      |  |

## *Part 2: Graphical Method*

Using your graph paper, draw vectors that represent each of the two forces from Part 2 along with their resultant force. Graphically determine the magnitude of the resultant in grams and its direction in degrees. Record the results in the table below.

| Graphical Determination |       |      |        |
|-------------------------|-------|------|--------|
| Force                   | Angle | Mass | Length |
| 1                       |       |      |        |
| 2                       |       |      |        |
| Resultant               |       |      |        |

## Part 2: Component Method

Using the component method, determine the magnitude and direction of the forces described in Part 2 and analytically determine the magnitude and direction of their resultant. Record your results in the table below.

| Component Determination |           |           |               |               |
|-------------------------|-----------|-----------|---------------|---------------|
| Vector                  | Magnitude | Direction | X - Component | Y - Component |
| 1                       |           |           |               |               |
| 2                       |           |           |               |               |
|                         |           |           | $R_x =$       | $R_y =$       |

Resultant  $R =$    $\theta_R =$

## Part 2: Percent Error Calculations:

Calculate the percent error of the the experimental value of  $F_R$  compared to the calculated value of  $F_R$  (the accepted value). Also calculate the percent error of the graphical solution of  $F_R$  compared to the calculated solution.

$$\text{Experimental Error} = \left| \frac{\text{experimental value} - \text{calculated value}}{\text{calculated value}} \right| * 100\%$$

$$\text{Graphical Error} = \left| \frac{\text{graphical value} - \text{calculated value}}{\text{calculated value}} \right| * 100\%$$

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### *Part 3: Experimental Method*

Using the force table, simulate the forces described in Part 3 and experimentally determine the magnitude and direction of their resultant. Record your results in the table below.

| Experimental Determination |       |      |  |
|----------------------------|-------|------|--|
| Force                      | Angle | Mass |  |
| 1                          |       |      |  |
| 2                          |       |      |  |
| Equilibrant                |       |      |  |
| Resultant                  |       |      |  |

### *Part 3: Graphical Method*

Using your graph paper, draw vectors that represent each of the two forces from Part 3 along with their resultant force. Graphically determine the magnitude of the resultant in grams and its direction in degrees. Record the results in the table below.

| Graphical Determination |       |      |        |
|-------------------------|-------|------|--------|
| Force                   | Angle | Mass | Length |
| 1                       |       |      |        |
| 2                       |       |      |        |
| Resultant               |       |      |        |

### Part 3: Component Method

Using the component method, determine the magnitude and direction of the forces described in Part 3 and analytically determine the magnitude and direction of their resultant. Record your results in the table below.

| Component Determination |           |           |                  |                  |
|-------------------------|-----------|-----------|------------------|------------------|
| Vector                  | Magnitude | Direction | X - Component    | Y - Component    |
| 1                       |           |           |                  |                  |
| 2                       |           |           |                  |                  |
|                         |           |           | R <sub>x</sub> = | R <sub>y</sub> = |

Resultant  $R =$    $\theta_R =$

### Part 3: Percent Error Calculations:

Calculate the percent error of the the experimental value of  $F_R$  compared to the calculated value of  $F_R$  (the accepted value). Also calculate the percent error of the graphical solution of  $F_R$  compared to the calculated solution.

$$\text{Experimental Error} = \left| \frac{\text{experimental value} - \text{calculated value}}{\text{calculated value}} \right| * 100\%$$

$$\text{Graphical Error} = \left| \frac{\text{graphical value} - \text{calculated value}}{\text{calculated value}} \right| * 100\%$$

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### *Part 4: Experimental Method*

Using the force table, simulate the forces described in Part 4 and experimentally determine the magnitude and direction of their resultant. Record your results in the table below.

| Experimental Determination |       |      |  |
|----------------------------|-------|------|--|
| Force                      | Angle | Mass |  |
| 1                          |       |      |  |
| 2                          |       |      |  |
| Equilibrant                |       |      |  |
| Resultant                  |       |      |  |

### *Part 4: Graphical Method*

Using your graph paper, draw vectors that represent each of the two forces from Part 4 along with their resultant force. Graphically determine the magnitude of the resultant in grams and its direction in degrees. Record the results in the table below.

| Graphical Determination |       |      |        |
|-------------------------|-------|------|--------|
| Force                   | Angle | Mass | Length |
| 1                       |       |      |        |
| 2                       |       |      |        |
| Resultant               |       |      |        |

### Part 4: Component Method

Using the component method, determine the magnitude and direction of the forces described in Part 4 and analytically determine the magnitude and direction of their resultant. Record your results in the table below.

| Component Determination |           |           |                  |                  |
|-------------------------|-----------|-----------|------------------|------------------|
| Vector                  | Magnitude | Direction | X - Component    | Y - Component    |
| 1                       |           |           |                  |                  |
| 2                       |           |           |                  |                  |
|                         |           |           | R <sub>x</sub> = | R <sub>y</sub> = |

Resultant  $R =$    $\theta_R =$

### Part 4: Percent Error Calculations:

Calculate the percent error of the the experimental value of  $F_R$  compared to the calculated value of  $F_R$  (the accepted value). Also calculate the percent error of the graphical solution of  $F_R$  compared to the calculated solution.

$$\text{Experimental Error} = \left| \frac{\text{experimental value} - \text{calculated value}}{\text{calculated value}} \right| * 100\%$$

$$\text{Graphical Error} = \left| \frac{\text{graphical value} - \text{calculated value}}{\text{calculated value}} \right| * 100\%$$

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When you have completed all four parts of this experiment, you may summarize your results in the data table below for inclusion in the Data section of your lab report. The shaded columns indicate which sample calculations should be performed.

*(Note: The % Error refers to the percent error in the magnitude of the resultant. You don't need to calculate a percent error for the angle  $\theta$ .)*

You should also attach all four of the graphs that were used for the graphical determination.

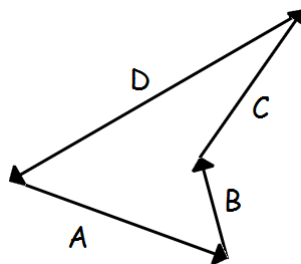
## Data Summary Table

|         |      |          |      |           | Resultant Determination Method |              |      |          |           |      |          |         |
|---------|------|----------|------|-----------|--------------------------------|--------------|------|----------|-----------|------|----------|---------|
| Force A |      | Force B  |      | Component |                                | Experimental |      |          | Graphical |      |          |         |
|         | Mass | $\theta$ | Mass | $\theta$  | Mass                           | $\theta$     | Mass | $\theta$ | % Error   | Mass | $\theta$ | % Error |
| 1       |      |          |      |           |                                |              |      |          |           |      |          |         |
| 2       |      |          |      |           |                                |              |      |          |           |      |          |         |
| 3       |      |          |      |           |                                |              |      |          |           |      |          |         |
| 4       |      |          |      |           |                                |              |      |          |           |      |          |         |

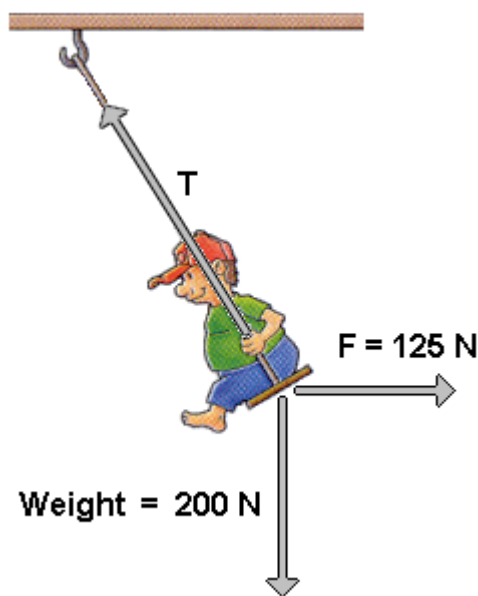
### A Few Questions

1. Suppose three forces, each having different magnitudes, are in equilibrium on the force table. Will they still be in equilibrium if you add an equal amount of mass to all three of the weight hangers? *Why?* (Hint: Use the force table and your setup from Part 1 to justify your answer. After establishing equilibrium, add 50 gm to all three weight hangers and describe what happens.)

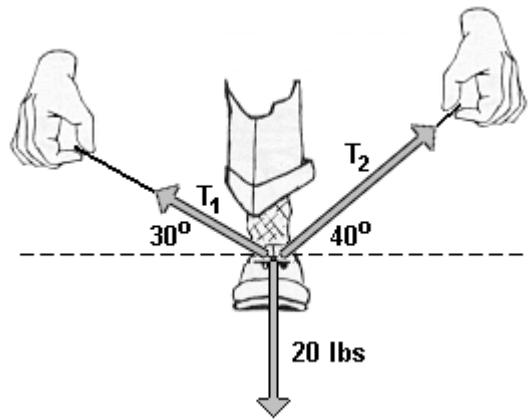
2. If **D** is the vector that balances the effect of vectors **A**, **B**, and **C** all acting together, which vector will balance the effect of vectors **A**, **C**, and **D** all acting together? (Note: vectors **A**, **B**, **C**, and **D** are in equilibrium.) *Explain your answer.*



3. A child weighing 200 N is being held back in a swing by a horizontal force of 125 N. *Use the graphical method* to determine the tension  $T$  in the rope and the angle  $\theta$  that the rope makes with respect to the horizontal. Include the graph in your lab report. *Verify your answer using the force table.*



4. While tying his shoes, a person holds the shoestrings at the angles indicated in the illustration below and pushes downward with a force of 20 lbs. *Use the component method* to determine the tensions  $T_1$  and  $T_2$  in each shoestring. *Verify your answer using the force table.*



5. The two ropes supporting the hammock shown in the illustration below are both tied to the trees at the same height. At what angle  $\theta$  will the tension  $T$  in each rope be equal to the person's weight. In other words, at what angle  $\theta$  does ***Tension = mg***?

(Hint: Use the force table to find the angle  $\theta$  by placing three equal masses on the three strings. Position one of the masses (representing the person in the hammock) at  $270^\circ$  and position the other two masses (representing the tension in the ropes) at the angles that establish equilibrium.) *Cross check your answer by finding the angle  $\theta$  using the component method.*

